Logic, Language and Modularity

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Modularity of Meaning I
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*Material from Fox and Hackl (2006)

Introductory Remarks

A Very Basic Question

What are the formal foundations of Human Reasoning?

In particular…

1. **Logic**:

   Is there such a thing as natural logic, a component of the mind/brain that derives inferences that we might call *logical* or *formal* ?

   I.e., is there an internal *Deductive System, DS*?

2. **Language**:

   If so, what is the relationship between natural language and DS?

3. **Modularity**:

   How do we distinguish inferences derived by DS from inferences derived with the help of other cognitive systems?

General Claims:

1. **Logic**
   Yes, DS exists.
2. **Language**
   DS is a component of the linguistic system
3. **Modularity**
   DS is "informationally encapsulated"
   it derives inferences from sentences based on *formal properties alone*.

Some Specific Claims

1. **Aspects of arithmetic are part of DS**: DS derives inferences pertaining to arithmetic notions: degrees and scales.

2. **Density**: However, the relevant notion of degree is characterized by the axioms of densely ordered domains. Cardinality (natural number) does not appear to be a notion of DS.

3. **Modularity:** Cardinality does contribute to meaning, but this contribution results from extra-linguistic interactions.

The Evidence for DS and Modularity

Certain rules of grammar show

a. **Sensitivity** to patterns of logical inference.
b. **Blindness** to non-logical inference.

Examples…

1. **Polarity Licensing** (Fauconnier 1975, 1979, Ladusaw 1979,Kadmon and Landman 1993, and quite a bit of subsequent work)
2. **Scope Economy** (Fox 1995, 2000, Reinhart 2006, Takahashi 2003, Miyagawa 2006, Spector and Mayr 2009)
3. ***Contradiction**: A general constraint against logical contradictions and tautologies (Barwise and Cooper 1983, Chierchia 1984, 2005, von Fintel 1993, Gajewski 2002, passim)

Degree Constructions

Standard Assumption: Two different kind of scales

(1) a. **Discrete:**

John has more than 3 children.

$\exists n>3$: *John has n children.*

b. **Dense:**

John is more than 6 feet tall.

$\exists d>6$: *John is d feet tall.*

Goals for this Talk

1. To sketch an argument from Fox and Hackl (2006) that scales are always dense.

2. To explain why the claim must be accompanied by a strong modularity thesis – one which is supported on independent grounds.

The Universal Density of Measurement (UDM)

The Intuitive Claim: Scales of height, size, speed, and the like are dense.

The Radical Claim: All scales are dense; cardinality in not a concept of NLS.

The Radical Claim $\Rightarrow$ Strong Modularity

Illustration of the Argument

A constraint on *only* and on exhaustive meanings

Background: implicatures and their correspondence to sentences with *only*

(1) a. John has three children.

Implicature: $\neg\exists n>3$ [J. has n children].

b. John has very few children. He only has THREE.

(2) a. John weighs 150 pounds.

Implicature: $\neg\exists d>150$ [J. weighs d pounds].

b. John weighs very little. He only weighs 150.

Density as an Intuitive Property of Scales

The Basic Effect

(1)a. John weighs more than 150 pounds.

*Implicature:

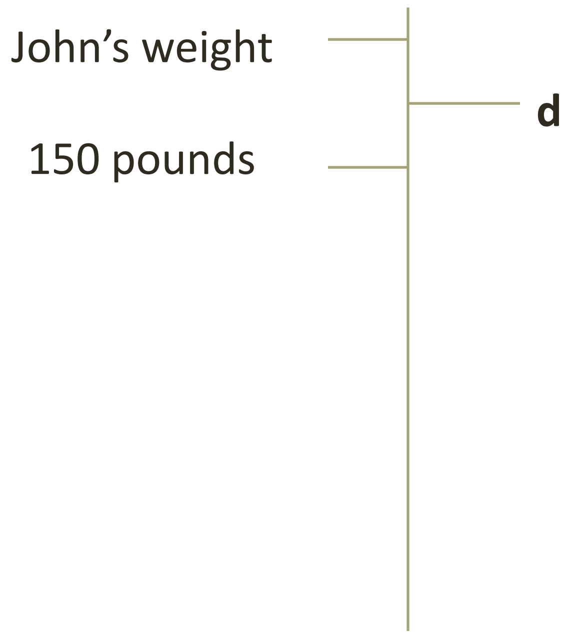
$\neg\exists d>150$ [J. weighs more than d pounds].

b. John weighs very little.

*He only weighs more than 150.

The Basic Effect – Picture

Density $\Rightarrow$150 can't possibly be the maximal degree that John's weight exceeds.



UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(2) a. You're required to weigh more than 300 pounds (if you want to participate in this fight).

Implicature:

$\neg\exists d>300$ [You're required to weigh more than d pounds].

b. You're only required to weigh more than 300 pounds.

EXISTENTIAL MODALS DO NOT

(3) a. You're allowed to weigh more than 150 pounds (and still participate in this fight).

*Implicature:

$\neg\exists d>150$ [You’re allowed to weigh more than d pounds].

b. *You're only allowed to weigh more than 150 pounds.

Density as a Formal Property

THE BASIC EFFECT

(1)a. John has more than 3 children.

*Implicature:

$\neg\exists d>3$ [J. has more than d children].

b. John has very few children.

*He only has more than 3.

Universal Modals Circumvent the Problem

(2) a. You're required to read more than 30 books.

Implicature:

$\neg\exists d>30$ [You're required to read more than d books].

b. You're only required to read more than 30 books.

EXISTENTIAL MODALS DO NOT

(3) a. You're allowed to smoke more than 30 cigarettes.

*Implicature:

$\neg\exists d>30$ [You're allowed to smoke more than d cigarettes].

b. *You're only allowed to smoke more than 30 cigarettes.

An obvious caveat

This was only the flavor of an argument.

To evaluate one would need to understand the entire paradigm. I.e., to consult the relevant literature:

Fox and Hackl (2006, *Linguistics and Philosophy*)

Fox (2007, *SALT Proceedings*)

Nouwen (2008, *Natural Language Semantics*)

Abrusan and Spector (2008, *WCCFL*, 2011, *Journal of Semantics*).

For further discussion see http://web.mit.edu/linguistics/people/faculty/fox/MIT_Colloq_2010.pdf

Modularity

A Problem

(1) a. I can say with certainty that John has more than 3 children.

Implicature:

$\neg\exists d>3$ [I can say with certainty that John has more than d children].

b. I can only say with certainty that John has more than 3_F children.

The truth conditions of these sentences seem to indicate that only integers count.

Towards a Restatement

There is a more basic problem:

the rounding/granularity problem.

(1) John is six feet tall

The meaning of a sentence is determined in a context which specifies (among many other things) a level of granularity, G.

Implementation

Granularity as an equivalence relation

Let C be a context in which G is the relevant level of granularity

(1) *John is exactly six feet tall*

Expresses in C the claim that John's height stands in the G relation to the degree *six ft*.

In short:

$$\text{Height}_{\text{feet}}(\text{J}) \in \text{Equivalence-Class}_{\text{G}}(6)$$

Likewise

(1) *John is exactly 15 years old*

Expresses in C the claim that John's age (in years) stands in the G relation to 15.

Given existing conventions:

$$Age_{years}(J) \in [15, 16)$$

Towards a Restatement

What we wanted to say:

(1) Only[John is more than 15_F years old]
Expresses the claim that:
i. $Age_{years}(J) > 15$
ii. $\forall d>15\ \neg[Age_{years}(J) > d]$.

$\Rightarrow$Contradiction (since the set of degrees is dense).

However, this line of reasoning ignores contextual parameters, and in particular, the granularity parameter G.

A Restatement of the Problem

Once G is taken into account, it is no longer obvious that the truth-conditions are contradictory:

(1) Only[John is more than 15_F years old]
Expresses the claim that:
i. $Age_{years}(J) > [15, 16)$
ii. $\forall EC > [15, 16) \neg [Age_{years}(J) > EC]$.
(where EC ranges over equivalence classes determined by G)

$\Rightarrow$**not contradictory.**

The Solution – Modularity

- G doesn't enter the picture at DS – the level at which *Contradiction is evaluated.
- G enters the picture with other extra-linguistic (a.k.a. *contextual*) aspects of meaning.
- At DS, sentences are ruled out if they can be proven to be contradictory under their context independent – *diagonalized* – meaning (equivalently, under the stringent granularity, *identity*)

Additional Evidence for the same Modularity Thesis:

Fox 2000

  for Scope Economy

Gajewski 2002, 2008, 2009

  for *Contradiction and NPI licensing

Magri 2006, 2008, 2011

  for the computation of Scalar Implicatures

Abrusan 2008

  for *Contradiction

Singh 2008

  for Maximize Presupposition

Conclusions

1. There is evidence that grammar rules out formal contradictions. If this is correct, it could be used to identify the formal vocabulary of natural language, and the rules of logical-syntax that characterize this vocabulary.

2. The semantics for the logical rules that characterize degree expressions involves dense degree domains.

3. Scalar implicatures are derived by a lexical item, a member of the logical vocabulary, *exh*.

Conclusions

4. Integers enter into the determination of truth conditions. However, this takes place within pragmatic system, via contextual parameters (integers are not part of what is sometimes called *narrow content*).

Conclusions

5. possible connection with work in experimental psychology

Recurring hypothesis: Prior to the development of adult arithmetic there is a core system that allows the measurement (or at least the estimation) of quantities, but crucially does not have access to anything like the notion of a natural number (Carey, Dehaene, Gelman and Gallistel, Spelke, among many others.) Perhaps the core system is the one relevant for NLS.

Appendices

*Contradiction and Modularity

*Contradiction -- Example #2

von Fintel (1993)

(1) a. Every man but John came to the party.

1. It is false that every man came to the party.
2. It is true that every man other than John came to the party.

b. *A man but John came to the party.

1. It is false that a man came to the party,
2. It is true that a man other than John came to the party.

$\Rightarrow$CONTRADICTION

*Contradiction -- Example #3

Dowty 1979:

(1) *John accomplished his mission for an hour.

There is a time interval in the past T s.t. Length(T) = one hour and

$\forall t \subseteq T$ John accomplished his mission in t.

And many, many other examples. See, e.g. , Lahiri 1998, Chierchia 2005, and Menéndez-Benito 2005.

But, some contradiction are acceptable

*Contradiction -- Example #3

(1) a. This table is both red and not red.
    b. He's an idiot and he isn't.
    c. I have a female (for a) father.
    d. I have 3.5 children.

(2) What you're saying is obviously false.
    a. It fallows that there is no man who arrived and yet that a man other than John arrived.
    b. #It fallows that a man but John arrived.

Gajewski's Conclusion

Nevertheless there is a general condition that disapproves of contradictions.

But the relevant system (DS) is modular: it is blind to the non-logical vocabulary.

Degree Relevant Theorems of DS

The UDM and DS

- If DS is thought of in syntactic terms (the terms of logical-syntax), then what are the axioms and rules of inference?

- If the arguments for the UDM are correct, the following seem to be required theorems

Theorems of DS

(1) a. Universal Density: $\forall d_{1,} d_2 \, [d_1 > d_2 \rightarrow \exists d_3(d_1 > d_3 > d_2)]$

b. Lexical Monotonicity: lexical n-place relations that have a degree argument are upward monotone (downward scalar).

c. Lexical Closed Intervals: if R is a lexical n-place relation, whose m^{th} argument is a degree, then for every *w*, and for every $x_{1,}\ldots,x_{n-1}$ $Max_{inf}(\lambda d.R(x_1)\ldots(d)\ldots(x_{n-1}))(w)$ is defined.

d. Commutativity: Two existential quantifiers can be commuted.

(23)d is also needed for the proposal in Fox (2000), where similar arguments for modularity are made.

Syntactic Contextual Restriction

Counter Examples to Negative Islands

Kroch (1989): When the context provides an explicit set of alternatives, negative islands are circumvented:

(1) Among the following, please tell me how many points Iverson didn't score?

a. 20 b. 30 c. 40 d. 50

What is the most informative degree in C, s.t. Iverson didn't score d points?

C = {20, 30, 40, 50}

Extends to Only and Implicatures

(1) Iverson sometimes scores more than 30 points. But today he only scored more than 20_F.

(2) A: How many points did Iverson score last night?

B: I don't know.

A: Was it more than 10, more than 20 or more than 30.

B: He scored more than 20 points

Implicature: he didn't score more than 30.

Exh/Only[C] [Iverson scored more than 20 points]

C= {that Iverson scored more than 10 points, that I. scored more than 30 points}